Strategies for Teaching Mathematics to Students with Dyslexia and Dyscalculia

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What does it mean to *know* mathematics?

Take a moment and just think about how you' d answer this question.

Share some of your thoughts with people nearby.

Now let' s talk aloud about things you heard or were thinking about.

Mathematics learning is complex

"Mathematics is more than completing sets of exercises or mimicking processes the teacher explains.

Doing **mathematics means generating strategies for solving problems, applying those approaches, seeing if they lead to solutions, and checking to see if your answer/s make sense."** (Van De Walle, 2010, pg. 13)

And *knowing mathematics* means being able (in some way) to communicate what you know to others.

This can be "tricky" for students with communication and/or memory difficulties as well as executive function challenges and diagnosed mathematics learning disabilities; **but it's far from impossible**.

Let's begin simply (and without a context).

Here's one addition and one subtraction expression. *Mentally* solve whichever one you feel most comfortable solving.

$$47 + 35 = \qquad\qquad 73 - 46 =$$

What is the mathematics that you needed to know in order to compute?

Did you also need to know what a reasonable answer might be?

If your sum was 712 would you have thought to yourself, "That can't be right!"

Or if the difference was 33 would you have thought, "Hmm, I thought it would be closer to 25."

Student's ability to decide on the reasonableness of a response is important for teachers to foster.

But – you will see in a short while that this is VERY difficult for students with dyscalculia.

However, meaningful tasks, relevant contexts, and teaching of concepts before procedures can enable many students with mathematics learning disabilities to be more successful than they would be without attention to these things.

Before we look at students with learning challenges, let' s

look at the most current ideas regarding what should be taught for <u>students to become mathematically powerful</u>.

Following decades of standards developed by the National Council of Teachers of Mathematics (NCTM)(1989, 1991, 1995, 2000, 2006), mathematicians, people in business and industry, as well as mathematics educators added their voice to *what* should be taught – and *how* math should be taught.

While currently increasingly "politicized", the *Common Core Standards for Mathematics* was disseminated in June 2010

"The need for coherent standards that promote college and career readiness has been endorsed across all states and provinces, whether or not they have adopted *CCSSM*" (NCTM, 2014, pg. 3)

The *CCSSM* provides a consistent, clear understanding of what students are **expected to learn**, so **teachers and parents** know what they need to do to help them.

The standards are designed to be **robust and relevant** to the real world, reflecting the knowledge and skills that young people need for **success in college and careers**.

There are TWO parts to the *CCSSM*

<u>Standards for Mathematical Practice</u> – these are the "habits of mind" that teachers help students develop.

- make sense of problems and persevere in solving them;
- reason abstractly and quantitatively;
- construct viable arguments and critique the reasoning of others;
- model with mathematics;
- use appropriate tools strategically;
- attend to precision;
- look for and make use of structure; and
- look for and express regularity in repeated reasoning

Along with the Standards for Mathematical Practice are the **Content Standards**

Depending on the grade of the student the DOMAINS may vary.

K-5 standards provide students with a solid foundation in whole numbers, computation with whole numbers, fractions and decimals and geometry.

With a strong K-5 foundation students will do "hands on learning in geometry, algebra, probability and statistics and will be well prepared for algebra in grade 8".

The middle school standards emphasize proportional reasoning, number systems, geometry and measurement.

The high school standards rely on the preparation that's taken place in middle school; asking students to apply mathematical ways of thinking to relevant world issues and challenges.

".. standards emphasize mathematical modeling, the use of mathematics and statistics to analyze empirical situations, understand them better, and improve decisions."

Knowing all of this, how can we, as educators, provide the

instruction that will enable all students to not only be successful, but to excel to the best of their ability?

And, how can this be done with students who have diagnosed learning differences that make receptive or expressive language challenging or who have been diagnosed with dyslexia or dyscalculia?

Dyscalculia and other mathematics learning difficulties have only recently received the attention and research that dyslexia has had.

The National Center for Learning Disabilities has expanded its "information base" to include mathematics learning disabilities and a roundtable of experts have been advising them on the research base. Their website, http://www.ncld.org has added mathematics and LD information. (Berch and Mazzocco, 2007)

National Center for Learning Disabilities' definition of dyscalculia

"Dyscalculia refers to a wide range of lifelong learning disabilities involving math. There is no single type of math disability. Dyscalculia can vary from person to person, and it affects people differently at different stages of life. Work-around strategies and accommodations help lessen the obstacles that dyscalculia presents. And just like in the area of reading, math LD is not a prescription for failure."

AND YET… definitions differ and researchers seem to differentiate between dyscalculia and mathematics learning disabilities.

Medical studies seem to have dominated both past and current investigations on these disabilities.

It was a Swedish neurologist, Salomon Henschen, in the 1920's who studied these "disorders" and coined the term *acalculia* (acquired disability of mathematics calculations) before the term *dyscalculia* (developmental mathematics disorders) was described – nearly 50 years later. (Berch and Mazzocco, 2007)

While definitions may vary, students with mathematics learning disabilities have similar characteristics.

- Less efficient working memory skills (poor memory for basic facts);
- Difficulty with number processing (Hanich et al, 2001; Landerl, Bevan, & Butterworth, 2004)
- Poor metacognitive skills – related to their performance (cannot explain whether their answer is correct or the strategies used to arrive at an answer);
- Slow, difficult and immature ways of dealing with numbers and computation;
- Difficulty with arithmetical algorithms; and
- Difficulty with applying properties of numbers

Without being too simplistic, research seems to indicate that **teaching to the standards** would enable more MLD students (and others) to be successful in mathematics.

This approach places greater demands on student's executive function skills but allows students to make sense of the concepts being taught without needing to memorize procedures.

A meaningful, problem-solving approach *can* be implemented if teachers know how to "systematically scaffold" concepts and skills.

(Roditi and Steinberg, 2007, 239)

Blair and Razza (2007) note the following

"Proficiency in mathematics, at all levels, requires the individual to reason actively about problem elements when arriving at possible solutions. The problem solving process requires the individual to represent information in working memory, to shift attention appropriately between problem elements, and to inhibit the tendency to respond to only the most salient or most recent aspect of a given problem." (2007, pp. 658-659)

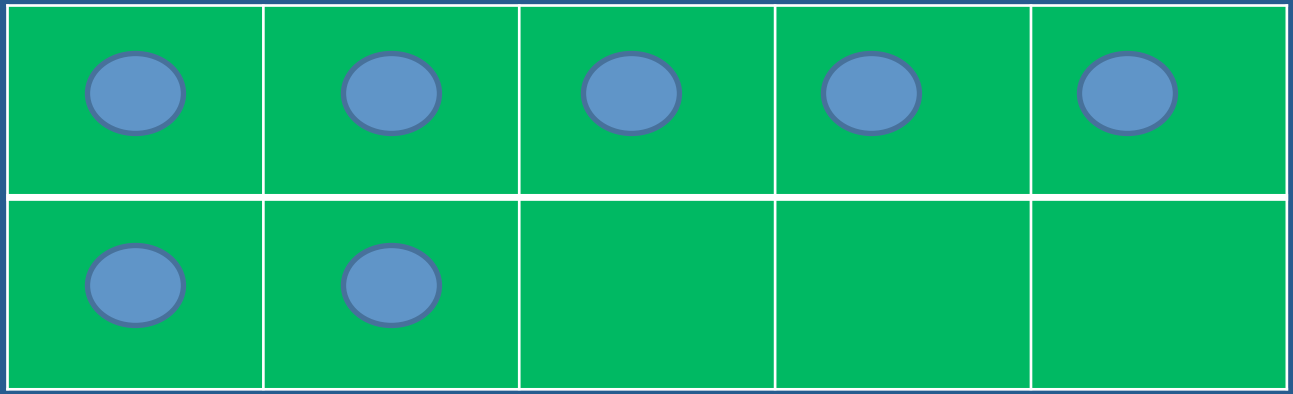
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Tasks like this one, and others, allow students to see that there are many “right” answers to problems asked.

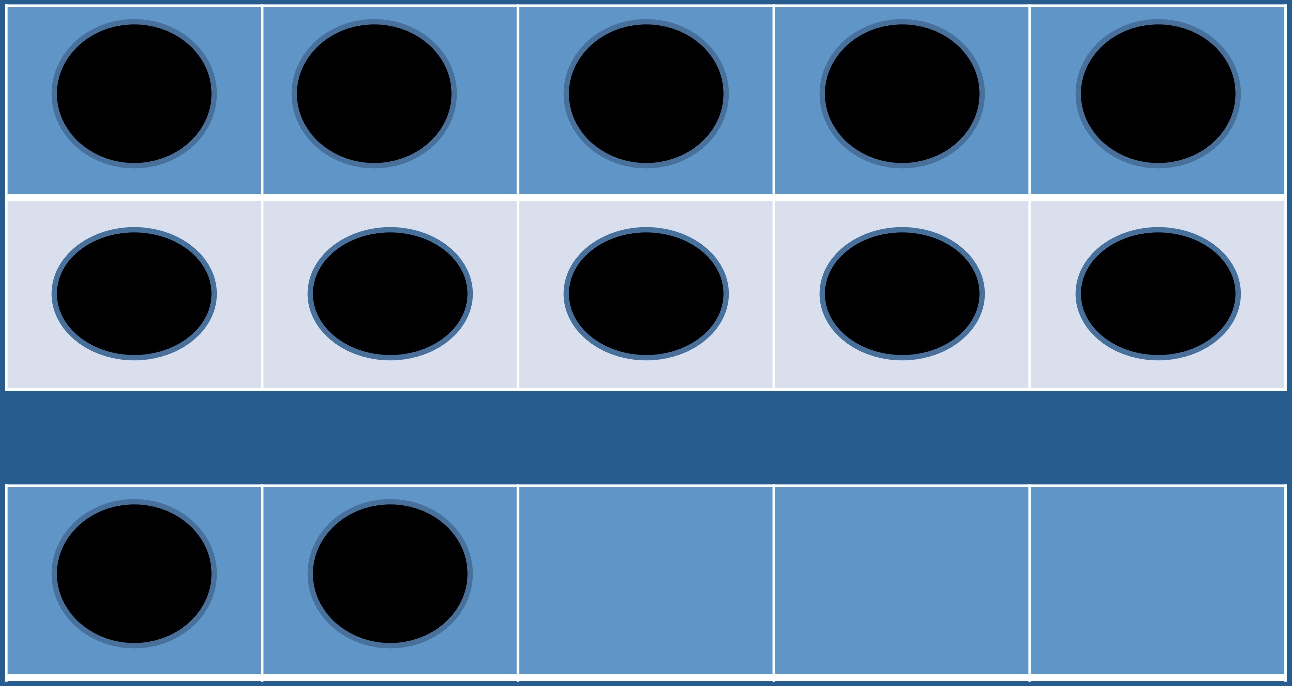
Younger students can/should begin with simpler arrangements and be asked to tell what they notice.

Once "five-frames" have been used to reinforce the quantities of 0 - 5

Ten-frames provide students with opportunities to practice seeing larger quantities. Repetitive games with these cards help students *subitize* arrangements so that they no longer need to "unit count".



As students develop mastery with quantities between 0 and 10 these same materials can assist in developing fluency with teen numbers.



Scaffolding, in this way, provides practice and is developmentally appropriate.

Activities of this sort work on student's:

- ❖difficulty with number processing as well as
- ❖ their immature ways of dealing with numbers.

Rather than "counting by ones" (which students realize will get them the correct answer), they begin to see the same quantities again and again and then trust that they can *count on* from one quantity to determine the total amount.

Benefits of instruction based on the *Standards*

Standards-based mathematics instruction enables educators to begin with what students know and build new understandings based on this background knowledge.

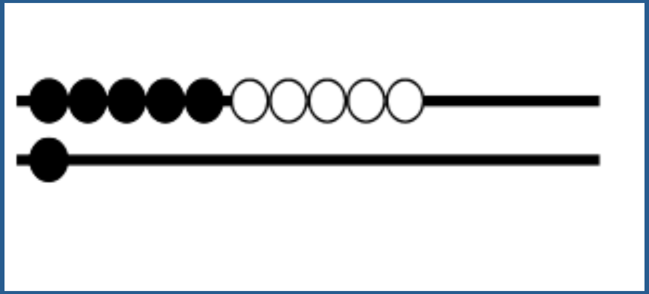
Griffin & Case (1997) explain that many commercial mathematics programs put mathematical demands on students before they are ready to "tackle" them. Formal symbolism, premature computation, and other abstract concepts need to be approached when students are ready.

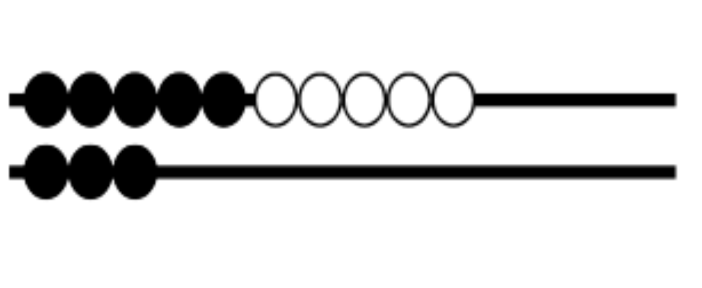
Following repeated practice with subitizing cards students can begin working with decomposing quantities

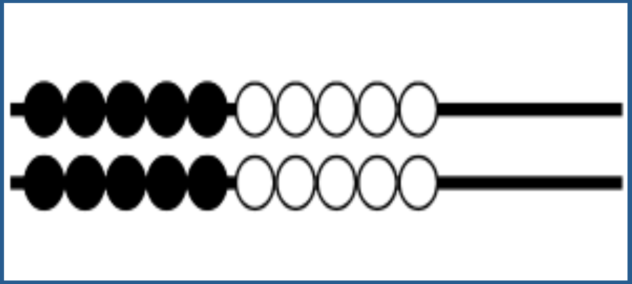
Rekenreks® provide practice with ways to represent different quantities in a very tactile-kinesthetic manner.

"Designed by Adrian Treffers, a mathematics curriculum researcher at the Freudenthal Institute in Holland, this material supports the natural development of number sense in children. Smaller versions consist of two rows of 10 beads. Larger versions with ten rows of ten beads are also available. Each row is made of five red beads and five white beads. This allows students to make mental images of numbers. Using 5 and 10 as anchors for counting, adding and subtracting is obviously more efficient than one-by-one counting."

Students begin to use 5 and 10 as anchor numbers







Other mathematics manipulatives provide multiple representations which helps to build understanding

As children toss bi-colored discs to find different ways to represent "6" (as 3 red and 3 yellow or 2 red and 4 yellow or 1 red and 5 yellow, ...) they begin to build a conceptual understanding of how a quantity can be decomposed and then composed again.

Cuisenaire Rods, connecting cubes, dominoes, and even dice can also be used to reinforce these concepts.

Following many experiences with part/whole ideas for quantities children are ready for simple computation

There is NO NEED yet for formal symbolism. Meaningful story problems – written and read orally, provide the background for students to make sense of computation ideas.

As long as manipulative materials are available; and children are also encouraged to 1) draw pictures, 2) use an open number line, or talk aloud about what they' d do to solve the problem, problem solving need not so difficult.

Number Talks (Parrish, 2010) provide opportunities to build student's number sense

They are short (5 – 15 minute) "conversations" about computation problems that STRENGTHEN student's **accuracy, efficiency and flexibility** with mental computation.

These can lead into the mathematics lesson, be done with small groups of students, or even end a mathematics lesson.

There are 5 key components to conducting a *Number Talk*

1. **A classroom culture needs to be established where ideas are accepted and student's responses are respected.**
2. **The classroom discussion provides students with opportunities to share their strategies and justify their thinking.**
3. **The teacher's role is to provide the problem, give "wait time", record answers, and facilitate the discussion.**
4. **Student's mental computation encourages them to build number relationships and not just rely on memorized procedures.**
5. **Problems are chosen purposefully to give students practice with specific mental computation strategies.**

Let's do some examples from Kindergarten through grade 5

Starting with **pre-kindergarten/kindergarten**:

SUBITIZING CARDS

First Grade: 8 + 6 =

Second Grade: 69 - 13 =

Third Grade: 18 + 4 + 18 =

Fourth Grade: 9 X 24 =

Fifth Grade: 12 X 49 =

"Noticings" and "Wonderings"

Too often we limit what we ask students to do, if they've been diagnosed with a mathematics learning disability.
Instead we need to have high expectations and provide meaningful mathematics experiences.

Computational Fluency means: EFFICIENT, EFFECTIVE, and FLEXIBLE

Let students use strategies that are natural for them as long as they need these strategies; at the same time, expose them to a range of other effective problem-solving strategies (Griffin, 2003).

Specific learning challenges yield specific weaknesses in mathematics

Children who have difficulty attending will have gaps in their mathematics skills ("I was absent when we learned division."

Students who have difficulty with planning will have difficulties coming up with strategies to solve word problems.

Students with difficulty controlling their impulses will "attack" new content quickly and become frustrated easily.

Learned helplessness and passive learning often are a direct result of many of these behaviors.

Mathematics difficulties linked to areas of executive dysfunction

Executive dysfunction	What this might look like in mathematics class
Attention deficits	Ignore signs when computing, misread word problems, place value misalignment (including decimals), gap in skills/concepts
Working memory deficits	Steps to procedures not followed, inability to recall all elements of a problem, choosing incorrect strategy or algorithm to solve a problem
Planning deficits	Not leaving enough space on paper to solve problem, lots of erasers, selecting wrong algorithm

Cognitive Flexibility	**Difficulty with multistep problems, shifting among algorithms, comfortably moving from one concept to the next**
Self-monitoring	Failure to check work when it's completed, failure to note careless errors or notice if a solution makes sense, "copying" problems
Retrieval deficits	Slower retrieval of basic facts, and procedures
Frustration/tolerance	Gives up quickly, poor coping skills when something new is presented

“Contrary to popular opinion that students with attention deficits cannot attend, students with true attention deficits actually attend to too much – a multitude of sensory stimulation catches their attention” (Allsopp, Kyger, Lovin, 2007, pg. 52)

Due to the inability to filter out distractions students might 1) miss important information presented in class, and 2) focus on unimportant events taking place during class.

WHAT TO DO

* Teach procedures only after concepts have been presented so that missing ideas make more sense; memory is not what’s being relied upon

HOW DOES THIS LOOK:

Let’s take the complicated idea of **exponents.**

Students “try” to remember that the exponent “tells you how many times to ‘take’ the base number and multiply”.

But, OFTEN 3^2 ends up being 3 X 2.

What if instead students used square tiles and represented arrays for 1 X 1, 2 X 2, 3 X 3, 4 X 4, 5 X 5, …

As they model these arrays they record the product.

A discussion pursued and questions were asked about the shape that’s made each time. A term is GIVEN: square numbers and a concept is linked to the phrase.

Wouldn't students like math more if they consistently "got it"?

And, just think how easy three cubed would be if students actually got to build shapes that were : 1X1X1

2X2X2

3X3X3

Multilink cubes (which connect from all six faces) allow this and counting those cubes or looking at the layers enables students to see the product.

Consistently teach things in a meaningful context

For students in the younger grades (kindergarten through grade three/four) daily routines that build calendar (measurement) concepts, place value concepts, and help to develop number sense are critical.

For older students routines can still be established – these reinforce concepts learned

We' ve been in school for 45 days.

What are the factors of 45?

Is 45 even or odd?

Is 45 prime or composite?

How would you show 45¢ using coins?

What number is 10 more, 10 less, 100 more than 45?

Would 45 cubes fill this (hold it up) container?

If 45 were the number of hours, how many days would this be?

And, for even older students

Is 45 a square number? How could you prove this?

What would the square root of 45 be (about)? How could you model this?

What would ½ of 45 be? ¼?

What are all of the rectangular arrays that 45 squares could be arranged in?

What is 40 less than 45? What is 50 less than 45?

Memory Difficulties

Without visual, auditory, kinesthetic, or tactile cuing and supports – explicitly tied to concepts students may have difficulty communicating their mathematical understandings.

This may be tied in more to how meaningfully students have stored information into their memory and the strategies they have for retrieving it than to information storage problems.

Difficulty with retrieval is more closely linked to problems with organization and association.

Visual organizers and multiple opportunities to represent problems can help with these memory deficits.

Templates aid students in remembering steps to algorithms.

Number lines are critical to use in helping students learn concepts of greater than and less than, numbers that come between, rounding numbers to the nearest ____, and even performing simple computation.

Some computer programs provide adaptive instruction and then independent practice

Explicitly teach students how to highlight important information in a problem.

But, **DO NOT TEACH "KEY WORDS"**. If meaning and understanding is what you are after teaching key words just has students looking for the words and finding the numbers. KEY WORDS confuse.

<u>**Example**</u>: Altogether there are 46 students in fourth-grade. 29 of them are girls and the rest are boys. How many boys are there in fourth-grade?

Have students keep a Math-Strategies Notebook

For those who can write and illustrate their own strategies, definitions, and formulas have them do this. For those who cannot, provide them with cards for students to keep in a binder.

Keep a **WORD WALL** in the classroom with words discussed in class. Refer to them regularly. Color code them if necessary (geometry words in blue, fraction words in green, …

Self-Monitoring

Make older students aware of places where they frequently make errors. Remind them – with "cards" so they check to see that they haven't made these again.

Really work on ESTIMATION SKILLS and looking at the REASONABLENESS of an answer.

Students resist returning to a problem – often because it was so hard to solve it in the first place. Give them opportunities to do this with engaging problems.

ACRONYM for Self-Checking

Meltzer, et al. (2006)

P: change color of the pen/pencil

O: check operations

U: underline the question, directions, or information that's important.

N: Check the numbers.

C: Check calculations.

E: Was the answer close to your estimate.

How to Impact Success

Early intervention seems to be the common theme expressed by many researchers and educators (Ginsburg, 2006).

Not only does early intervention enable students with specific learning disabilities to develop foundational understandings; but they begin teaching students self regulation and metacognition skills that will be needed in later years.

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